

Local de Broglie-Bohm Trajectories from Entangled Wavefunctions

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We present a local interpretation of what is usually considered to be a nonlocal de Broglie-Bohm trajectory prescription for an entangled singlet state of massive particles. After reviewing various meanings of the term “nonlocal”, we show that by using appropriately retarded wavefunctions (i.e., the locality loophole) this local model can violate Bell’s inequality, without making any appeal to detector inefficiencies.

We analyze a possible experimental configuration appropriate to massive two-particle singlet wavefunctions and find that as long as the particles are not ultra-relativistic, a locality loophole exists and Dirac wave(s) can propagate from Alice or Bob’s changing magnetic field, through space, to the other detector, arriving before the particle and thereby allowing a local interpretation to the 2-particle de Broglie-Bohm trajectories.

We also propose a physical effect due to changing magnetic fields in a Stern-Gerlach EPR setup that will throw away events and create a detector loophole in otherwise perfectly efficient detectors, an effect that is only significant for near-luminal particles that might otherwise close the locality loophole.

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I. INTRODUCTION

In the EPR literature, it is usually considered that the only local models that can explain EPR experiments are contrived intellectual diversions^{1,2,3}, bearing no relation to physical reality, and that additionally require some “unfair” sampling mechanism to generate the correct experimental results^{4,5,6,7,8}. Somewhat less contrived, Accardi⁹ has proposed a chameleon model that is claimed to be local and violate Bell’s inequalities, and Christian¹⁰ had proposed a local model based on properties of a Clifford algebra. With the exception of the last two, most local hidden variable models assume a “detector” loophole.

It also seems to be conceded that – for a single massive particle³² – the de Broglie-Bohm interpretation of quantum mechanics is local, in that the gradients of the wave function at the particle position provide a quantum field that exerts a force on a deterministic, otherwise classical, particle. There seems to be general agreement that when it comes to two entangled particles, the same de Broglie-Bohm interpretation is manifestly nonlocal^{11,12}. It is with this argument that we take issue, and propose to regard the two-particle de Broglie-Bohm relations¹³ as a *local* model – which we claim can be done as long as time-retarded wavefunctions are used. We motivate this by an example: an analysis of Hooke’s Law and its apparent nonlocality.

We will show that our local de Broglie-Bohm model can violate Bell’s inequality as long as it can use the “locality” loophole. Since QM (Dirac) waves move at a phase velocity of c , and since massive particles move much slower, this loophole appears to be big enough for all conceivable experiments involving massive particles. We will also show that there exists a physical mechanism that could operate on these massive particles to provide a “detection” loophole, which we will see is not significant unless the particles are moving at ultra-relativistic speeds – but for such Stern-Gerlach experiments, means that one could still measure apparent violations of Bell’s inequality with a local mechanism.

II. HOW NON-LOCAL IS DE BROGLIE-BOHM?

In the single particle case, the de Broglie-Bohm velocity law takes the form

$$\vec{v}(x, t) = -\frac{i\hbar}{2m} \left(\frac{\psi^*(x, t) \nabla \psi(x, t) - (\nabla \psi(x, t))^* \psi(x, t)}{\psi^*(x, t) \psi(x, t)} \right) = \frac{\hbar}{m} \Im \left(\frac{\nabla \psi(x, t)}{\psi(x, t)} \right), \quad (1)$$

where $\psi(x, t)$ is the solution to the Schrödinger equation. This is a non-linear, but not obviously non-local, prescription for the velocity of the particle. If a boundary condition changes far away (e.g., closing a slit at $x = -L$ at time $t = 0$),

and the effect is felt at $x = +L$ simultaneously, we will claim that the nonlocality is a result of using the non-relativistic Schrödinger equation; using the relativistic Dirac equation means that the wavefunction at $x = +L$ will not change until after the time $\Delta t = 2L/c$, when the “Dirac wave” has propagated to that position (the velocity prescription for Dirac wavefunctions¹⁴ is similar to that of de Broglie-Bohm’s for Schrödinger wavefunctions: $dx^i/dt = cj^i/j^0$ where $j^\mu = c\bar{\psi}\gamma^\mu\psi$). Thus, for a single particle, we claim that de Broglie-Bohm interpretation is not *essentially* non-local.

In the case of two particles, we have

$$\begin{aligned}\vec{v}_1(t) &\sim \frac{\hbar}{m} \Im \left(\frac{\nabla_1 \psi(x_1, x_2, t)}{\psi(x_1, x_2, t)} \right) = v_1(x_1, x_2, t) \neq v(x_1, t) , \\ \vec{v}_2(t) &\sim \frac{\hbar}{m} \Im \left(\frac{\nabla_2 \psi(x_1, x_2, t)}{\psi(x_1, x_2, t)} \right) = v_2(x_1, x_2, t) \neq v(x_2, t) ,\end{aligned}$$

for the velocity of each particle, and see that, formally at least, neither particle’s velocity is strictly a function of it’s own position, as one might expect for classical particles in local potentials. This dependence is taken as *a priori* evidence of nonlocality of the de Broglie-Bohm interpretation^{16?}. We shall argue below that this dependence is misinterpreted.

Bohm’s justification of the physicality of nonlocality (*i.e.*, the transmission of forces at infinite speed) was to argue that it is a “non-signaling” non-locality, since his interpretation had been shown to lead¹¹ (p. 187) to “precisely the same predictions for all physical processes as are obtained from the usual interpretation (which is known to be consistent with relativity).” In other words, by taking the ensemble average of lots of non-local results he gets a local result.

We would prefer the grammatically simpler “locality” to the double negative of a non-signaling non-locality. As far as we can see, the only reason most physicists don’t agree, is that Bell has said that certain experimental measurements are evidence of nonlocality. We have argued elsewhere^{17,18} that Bell’s derivation assumed a certain non-contextuality that was too strict, and that a weaker limit could be derived for hidden variables that are “locally” contextual (by local contextuality we mean hidden variables that prevent Alice from measuring a' at the same time she measures a). This paper presents an example of a local hidden variable theory that is also “remotely” contextual, in that Alice’s result depends on her causal knowledge of Bob’s detector setting (*i.e.*, at a retarded time, $A(a(t), b(t - D/c), \lambda(t))$).

A. How non-local is Hooke’s Law?

Consideration of a wider relevant context prompts one to ask “How non-local is Hooke’s Law?”, because in that case we also have a force law that looks “manifestly” nonlocal:

$$m_1 \ddot{x}_1 = -k(x_1 - x_2) , \tag{2a}$$

$$m_2 \ddot{x}_2 = +k(x_1 - x_2) . \tag{2b}$$

We are not aware of any textbooks that use this as an example of non-locality (springs being an archetypical example of local, causal wave propagation), and most physicists would expect that even if m_2 were suddenly fixed (in Equation (2a)), that m_1 wouldn’t react to that event until a sound wave had passed down the spring from m_2 to m_1 . That being the case, perhaps x_2 should be retarded in the first equation and x_1 in the second; unless it is invalid to use these equations in that context.

The de Broglie-Bohm interpretation has been modified in precisely that manner^{15,19?} – to use time-retarded positions in a time-independent wavefunction – in order to force a local interpretation out of the velocity prescriptions. This means, of course, that if the original velocity prescription came from nonlocal non-signaling interpretations of the Dirac equation, that the new interpretation would be the solution to something that isn’t quite the Dirac equation anymore, and sooner or later, for some measurement this new re-interpretation will have to give a theoretical prediction different from the Dirac equation (thereby falsifying this retarded solution).

If the positions in Hooke’s law are retarded, we would then have

$$\begin{aligned}m_1 \ddot{x}_1(t) &= -k(x_1(t) - x_2(t - \tau)) , \\ m_2 \ddot{x}_2(t) &= +k(x_1(t - \tau) - x_2(t)) ,\end{aligned}$$

where $\tau = \Delta x/c_s$, and Δx is some measure of the separation. One can expand these new expressions about t for small τ , leading to

$$\begin{aligned}m_1 \ddot{x}_1(t) &= -k(x_1(t) - x_2(t)) - \tau k \dot{x}_2(t) , \\ m_2 \ddot{x}_2(t) &= +k(x_1(t) - x_2(t)) - \tau k \dot{x}_1(t) ,\end{aligned}$$

which still has the force on the first particle depending on the instantaneous position of the second particle, and *vice versa*, but now a damping force ($\propto c^{-1}$) depends on the instantaneous velocity of the other particle as well. It is possible that this *reductio ad infinitum* might converge short of *absurdum*, but we would suggest that retarding something that is not nonlocal in the first place will just make the local equations wrong.

Our preferred interpretation of this case is to take seriously the notion that Equations (2) are an *idealization only appropriate to an isolated system*. If the system is isolated, then no one can grab one particle to initiate a nonlocal signal in the first place, and if they do, then the system is no longer isolated, the equations no longer apply, and there is no issue of whether invalid equations can signal or not.

Nonetheless, within the idealization, there is a way for m_2 in equation (2b) to show indifference to m_1 's fate for at least a while, and that would be if m_2 took seriously the notion that it was part of an isolated system. Suppose m_2 “knows” not only his current location, but has “calculated” (with Equation (2a)) where the coupled and equally *isolated* m_1 should be and thus what acceleration m_2 should be experiencing (similarly, m_1 calculates m_2 's location in order to update her acceleration, in both cases using initial conditions learnt from earlier causal contact). In that case, m_2 will continue to oscillate regardless of whether something grabs m_1 at a particular time or not. Of course, m_2 will continue to oscillate even after $\Delta x/c_s$, but now we know that our (local) mathematical model is in a regime it is not designed to handle (and we can predict when any discrepancies should occur); we would have to go beyond two isolated mass points to the continuum model of Navier-Stokes equations in order to have enough sophistication to propagate sound waves from changing boundary conditions at one location to other locations.

Note that we can also transform the equations into another form that suggests more locality and less computational capabilities for mass points:

$$\begin{aligned} m_1 \ddot{x}_1(t) &= -k(x_1(t) - x_2(t)) , \\ &= -2k(x_1(t) - X_{cm}) . \end{aligned}$$

As long as the center of mass of the isolated system behaves predictably ($X_{cm}(t) = X_0$, which can always be done after a Galilean boost if $\dot{X}_0 \neq 0$), the only non-parametric variable in the force on m_1 is the position of m_1 in the “force-field” of the center of mass, *i.e.*, $F_1 = F(k, X_0; x_1)$.

In this way we see that an isolated system of particles can be such that one particle can depend on the instantaneous position of another, without requiring anything but a local, causal force field at each point (which effectively calculates m_1 's position for m_2 and *vice versa*). Without the concept of a force-field, Newton was flummoxed less by the nonlocality than the action at a distance; with the concept of a quantum potential and its field extending through the aether, there is no problem of m_1 acting on m_2 from a distance, although the question of whether it does it instantaneously or not requires us to determine what an “isolated” system is.

We will suggest that in an EPR experiment, by knowing $x_1(0)$, $x_2(0)$, and Ψ , Alice can calculate both x_1 and x_2 given the de Broglie-Bohm velocity prescriptions, and similarly for Bob. For Hooke's law, the differential equations describing the motion are only correct as long as the system is undisturbed – any attempt to grab one of the mass points destroys the isolation and changes the boundary conditions of an underlying continuum model; for an EPR experiment, the Schrodinger wavefunctions provide valid trajectories as long as the boundary conditions (Alice and Bob's magnet settings) are unchanged – otherwise, we have to wait for the Dirac waves to re-equilibrate the new settings. We remind the reader that the usual method of calculating de Broglie-Bohm trajectories is to solve $i\hbar\dot{\psi} = H\psi$ in order to propagate a wave packet from its initial position to the final position, so that the derivatives of that wave-packet wavefunction can provide the pilot instructions to the individual point particles. If the boundary conditions change during that time, it is clear that a more fundamental theory must be invoked – for example the Dirac equation, which will propagate changes to the wavefunction at the speed of light. Even so, this still leaves the $x_1 - x_2$ type functional dependence, which can be worked around in the same way that Hooke's m_1 and m_2 performed redundant calculations, by assuming that individual particles know the (fixed) position of the center of mass. If a more fundamental approach is needed, one can then resort to second quantization and QED, the continuum analogs to Navier-Stokes.

As with springs, for two quantum-mechanical particles one can recast a Schrödinger equation using individual coordinates, $\psi(x_1, x_2)$, into one using center-of-mass and relative coordinates, $\Psi(X, x)$, instead. In the Stern-Gerlach case, there are some practical advantages working with two coordinate systems that are not aligned with respect to each other (where Bob's $\hat{x}_2 \parallel \hat{B}_{bob}$ and Alice's $\hat{x}_2 \parallel \hat{B}_{alice}$, but $\hat{B}_{bob} \nparallel \hat{B}_{alice}$), and since the singlet state's relative coordinate wavefunction still entangles spin up and down, there is no advantage to using relative coordinates for this problem. (*N.B.*: when Alice and Bob's magnets are at the same angle, so that the two \hat{x} 's are parallel, the relative coordinate formulation does allow one to see that the de Broglie-Bohm velocity law factors into a single function of \vec{x} , independent of \vec{X} , shown in Equation (8) below. In other words, for aligned magnet settings, the de Broglie-Bohm velocity law takes on the same dependence on relative coordinates as a Hooke or Coulomb force law, which we have seen is local as long as the system is isolated.)

B. Entanglement and *a priori* Nonlocality

Rice²⁰ presents a geometric argument that the de Broglie-Bohm interpretation must be nonlocal, arguing, essentially, that an entangled wavefunction has nonlocality built into it despite (or in spite of) the local behavior of a single wavefunction. Consider the case of two non-interacting particles, such that $H_1\psi_1 = i\hbar\dot{\psi}_1$ and $H_2\psi_2 = i\hbar\dot{\psi}_2$. A basis-state wavefunction for the two-particle system would have the form $\psi_1(x_1, t)\psi_2(x_2, t)$, and an entangled, singlet state wavefunction can be written as $\psi_1(x_1, t)\psi_2(x_2, t) - \psi_1(x_2, t)\psi_2(x_1, t)$. It is this form that Rice claims is nonlocal: start by assuming that Bob makes some change to his boundary conditions at time $t = 0$. Alice (who we assume is at a distance D from Bob), in order to apply the de Broglie-Bohm method to describe the position of her particle (m_1), determines that her v_1 is proportional to $\psi_2(x_2, t)\nabla_1\psi_1(x_1, t) - \psi_1(x_2, t)\nabla_1\psi_2(x_1, t)$; if the time t is less than D/c , then no causal signal from Bob (*e.g.*, changes in his Dirac wavefunction) should have reached Alice. Nonetheless, the velocity that she calculates has a (first) term proportional to $\psi_2(x_2, t)$ – her velocity depends on Bob’s wavefunction (ψ_2) after his change to the boundary condition but before Alice can possibly know about it in any causal manner, even if Alice has been keeping track of where Bob’s m_2 is or should be currently located. In other words, it is not the $\psi(x_2)$ dependence that is non-local, it is the $\psi_2(t)$ that is non-local regardless of where x_2 is at. Whether these wavefunctions are solutions to Dirac or Schrödinger equations, it is this *form* of the entangled wavefunction that Rice claims is *a priori* nonlocal.

In regular quantum mechanics, the only use made of such entangled wavefunctions (or any wavefunction) is to calculate matrix elements, and the issue of nonlocality does not arise in such a case: one integrates over all dx_2 at a time t , resulting in something that is only a function of x_1 , which in turn is integrated over dx_1 to produce the number that is compared to some experimental measurement. (Note that during the integration over dx_2 and dx_1 , it is assumed that the wavefunction is static, corresponding exactly to an experiment done with detectors in fixed positions. If Alice and Bob are rapidly changing their settings, then the quantum-mechanical averages need to be done with more care than has yet been shown by anyone in the EPR community.) Only the de Broglie-Bohm interpretation makes use of the “pointwise” values of the wavefunction in its velocity prescriptions, and so only the de Broglie-Bohm interpretation suddenly confronts this peculiar form of nonlocality.

There are three possible solutions to this problem. The historical approach (Rice’s) is to assume that Alice is indeed godlike (or at least nonlocally omniscient), and instantaneously knows what Bob’s wavefunction has become, so that she can apply her nonlocal velocity algorithm to determine v_1 , and we again have a “non-signaling” nonlocality. Another approach (Squires’) is to retard the position of the opposite particle in the (presumably still nonlocal) time dependent wavefunction.³³

The alternative which we propose is to insist that Alice can only make use of locally available information, and that while she can use Bob’s wavefunction to calculate her velocity, it must be an appropriately retarded wavefunction – the one that Bob would have had if he had done nothing in the most recent D/c time interval. Unlike the classical Alice, who only had to calculate x_1 and x_2 from Hooke’s law, the quantum Alice not only has to calculate x_1 and x_2 , she has to calculate both ψ_1 and ψ_2 using only her causal information about all boundary conditions (ditto for quantum Bob).

It is hard to pick a good notation with which to write this; Alice will calculate a contemporaneous wavefunction, $\psi_2(x_2, t)$, to describe the velocity of Bob’s particle at the current time and non-retarded position, but this PDE will be solved using boundary conditions near Bob that are retarded to the time $t - D/c$; similarly, Bob will calculate $\psi_1(x_1, t)$, but using boundary conditions near Alice also retarded by D/c . We will refer to these as “retarded wavefunctions” as a shorthand for “contemporary wavefunctions based on retarded boundary conditions”, and write them as

$$\begin{aligned}\Psi_A(x_1, x_2, t) &= \psi_1(x_1, t)\psi_2(x_2, t - D/c) - \psi_1(x_2, t - D/c)\psi_2(x_1, t) , \\ \Psi_B(x_1, x_2, t) &= \psi_1(x_1, t - D/c)\psi_2(x_2, t) - \psi_1(x_2, t)\psi_2(x_1, t - D/c) .\end{aligned}$$

If the boundary conditions remain static, the wavefunctions evaluated with time-retarded boundary conditions become identical to ones with instantaneous boundary conditions, which means that Alice and Bob will then generate exactly the entangled results (that violate Bell’s inequality). If the boundary conditions do change, then it will take time for the new “entanglement wave” to propagate from Alice to Bob (or *vice versa*), and Bob, using “old” information, will calculate a trajectory different from one using non-local instantaneous information (we assume averages of such events will satisfy Bell’s inequality and give a concrete illustration of this in the section below on trajectories for massive singlets).

C. Nonlocality involving violation of Bell's Inequality

We^{17,18} and others²¹ have shown that Bell's derivation of Bell's inequality,

$$|\langle AB \rangle - \langle A'B' \rangle| + |\langle AB' \rangle + \langle A'B \rangle| \leq 2, \quad (3)$$

depended on a particular form of counterfactual definiteness – that two measurement *results* can be considered to exist at the same time – *e.g.* that position (A) and momentum (A') can be measured simultaneously – something that is forbidden by the Heisenberg Uncertainty Principle.³⁴

The “instruction set” paradigm of hidden variables⁷ can also be viewed this way: “the hidden variables are the results not measured” – not something that might be a proximate *cause* of a measurement (like a spin vector with some horizontal as well as vertical component, or what instruction sets *ought* to be), but the measurement result *itself*, since the instruction is plugged into the formula where the measured results are supposed to go. But what can a measurement result be in the absence of a measurement – the particle going left (unnoticed, in a virtual horizontal magnet?) while we detect that it is going up in our vertically oriented magnet? Clearly, if we cannot physically make the two measurements at the same time, we should not talk about the measurement results existing at the same time, even though such classical habits are hard to break.

Avoidance of such counterfactual reasoning results in a weaker inequality,

$$|\langle AB \rangle - \langle A'B' \rangle| + |\langle AB' \rangle + \langle A'B \rangle| \leq 2 + \min (|\langle B_1 B'_3 - B'_2 B_4 \rangle|, |\langle A_1 A'_4 - A'_2 A_3 \rangle|), \quad (4)$$

consistent with the fact that one measurement can only be made strictly before or after the other. This inequality also holds for any classical system for which two measurements cannot be made simultaneously.³⁵ Of course, this should not be a surprise, since one can easily show^{22,23} that the quantum-mechanical measurement operators (for 2-particle states) obey a similar inequality:

$$\hat{S}_{Bell}^2 = 4\hat{I} - [\hat{A}, \hat{A}'][\hat{B}, \hat{B}']. \quad (5)$$

A number of philosophical questions arise when we contemplate this relation: since there is no factor of \hbar multiplying the commutator terms on the right hand side of Equation 5, how is it that a putatively nonlocal quantum mechanics can reduce to a local classical mechanics. Furthermore, why is it only non-commutative experimental measurements that generate nonlocal results – if Alice switches from a to $a + 180^\circ$ ($A \rightarrow -A$), then even if Bob continues to switch between b and b' , nonlocality will not show up – what kind of nonlocality only shows up when measured at some angles but not others? The issues vanish if the explanation is that measurements are contextual, as shown, for example, by Christian's model¹⁰. As we are about to see, the issues will also vanish if Bell's assumption that Alice and Bob's measurements, made a distance D apart, can't be written as $A(a, \lambda)$ or $B(b, \lambda)$, but must be written as $A(a, b(t - D/c), \lambda)$ and $B(a(t - D/c), b, \lambda)$. All derivations of Bell inequalities begin by assuming the $A(a, \lambda)$ form as the mathematical form of the locality postulate, but this is strictly true only if $A(a(t), \lambda)$ has no dependence on $b(t)$; a dependence on $b(t - D/c)$, while local, is causally “contextual”. It is not clear if EPR would reject that out of hand given the alternative of true nonlocality.

III. DE BROGLIE-BOHM TRAJECTORIES FOR MASSIVE SINGLET

The de Broglie-Bohm analog of waiting for a sound-wave to reach the second mass point in a Hooke-law spring is to be understood, not by retarding the second particle's position, but by using the appropriately retarded wavefunctions in the velocity law (as we have argued in section II B). Imagine that Alice doesn't change her magnet, but Bob does. In that case, the wavefunction inside Bob's magnetic volume will change (adiabatically, we presume) while his magnet is being switched, and a Dirac wave (a wave in the electron aether distinct from any electro-magnetic Maxwell wave) will propagate out from that region at the speed of light toward Alice. As long as Alice uses retarded boundary conditions to calculate Bob's wavefunction, she can calculate both x_1 and x_2 in a causal manner for use in her velocity prescription for \dot{x}_1 ; similarly, if Bob uses his instantaneous wavefunction and retarded boundary conditions for Alice's wavefunction, his \dot{x}_2 will be local and causal.

Durt and Pierseaux¹³ defined an EPR experiment based, presumably, on typical Stern-Gerlach experiments. For their case of two silver atoms in a singlet spin state, they assumed that the velocity of the atom was 10^4 cm/s, and if we further assume that the overall size of the system is $\mathcal{O}(100$ cm), then Alice will know about Bob's new wavefunction in $\mathcal{O}(10^{-8}$ s), while the particles take $\mathcal{O}(10^{-2}$ s) to get from source to detector. Unless ultra-relativistic particles are

used, it would seem that there is no way for Alice *not* to know Bob's magnetic field setting in time to calculate a local expression for her particle.

The locality loophole will be very hard to close for massive particles, and de Broglie-Bohm velocity prescriptions using wavefunctions based on retarded boundary conditions will provide a local hidden variable theory that violates Bell's inequality in perfect agreement with the quantum results.

Holland¹⁶ (p. 469) and Durt & Piersaux¹³ have calculated the velocities of the two particles (using D&P's notation, which distinguishes Left from Right, but for which we can read Alice and Bob; with the local z axes parallel to the local magnetic fields, $\hat{z}_{L|R} \parallel \hat{B}_{L|R}$):

$$\frac{dz_L}{dt} = \frac{k^2}{1+k^2t^2}tz_L + \frac{s^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(z_L+z_R)}{2}\right) + c^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(z_L-z_R)}{2}\right) - c^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(-z_L+z_R)}{2}\right) - s^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(-z_L-z_R)}{2}\right)}{s^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(z_L+z_R)}{2}\right) + c^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(z_L-z_R)}{2}\right) + c^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(-z_L+z_R)}{2}\right) + s^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(-z_L-z_R)}{2}\right)} \cdot \alpha t \left[2 - \frac{k^2t^2}{1+k^2t^2} \right], \quad (6a)$$

$$\frac{dz_R}{dt} = \frac{k^2}{1+k^2t^2}tz_R + \frac{s^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(z_L+z_R)}{2}\right) - c^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(z_L-z_R)}{2}\right) + c^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(-z_L+z_R)}{2}\right) - s^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(-z_L-z_R)}{2}\right)}{s^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(z_L+z_R)}{2}\right) + c^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(z_L-z_R)}{2}\right) + c^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(-z_L+z_R)}{2}\right) + s^2e\left(\frac{\beta t^2}{1+k^2t^2}\frac{(-z_L-z_R)}{2}\right)} \cdot \alpha t \left[2 - \frac{k^2t^2}{1+k^2t^2} \right], \quad (6b)$$

where

$$\alpha = \frac{dB}{dz} \mu / 2m, \quad (7a)$$

$$\beta = 2\alpha / \delta r_0^2, \quad (7b)$$

$$k = \hbar / 2m\delta r_0^2, \quad (7c)$$

$$c \equiv \cos(\theta_A - \theta_B) / 2, \quad s \equiv \sin(\theta_A - \theta_B) / 2, \quad (7d)$$

with μ is the magnetic moment, m the mass of the silver atom, δr_0 the initial width of the Gaussian wavepacket and dB/dz the magnetic field gradient. θ_A and θ_B are the orientations of the magnetic fields with respect to a third, fixed, frame. When $s \neq 0$, the two z -axes are *not* aligned and the arguments of the exponentials are *not* the z -components of the relative and center-of-mass coordinates but a mixture of the z and x coordinates (assuming the particles separate from the source in the $\pm \hat{y}$ direction).

1. Local vs. Nonlocal Velocity Prescriptions

For Alice to retard Bob's boundary condition in her instantiation of the velocity expressions means that she must use a causal value for θ_B , and similarly, Bob must use only values for θ_A that he can know causally. Each then uses both equations to model both trajectories, using only the (jointly known) initial location of the particles.

For a nonlocal prescription, Alice is free to use the instantaneous value of Bob's magnet setting. Whether Alice continues to calculate both trajectories, or just "knows" Bob's instantaneous result is moot, since the two methods will result in the same answer.

Recall that for massive particles, it is almost certain that the Dirac wave from Bob's changing magnetic field will overtake the Alice-bound particle while it is in the field-free region between the source and Alice's magnet/detector. In that region, entangled Schrödinger wavefunctions generate the same simple velocity law as unentangled wavefunctions, $\vec{v}^A = -\vec{v}^B = -\hbar\vec{k}/m$. Thus, we hypothesize that the changes in the Dirac wave will not affect the particle as it passes over it, and that when the particle finally reaches Alice, she will know the correct magnetic field setting to use for Bob's boundary condition and therefore impose the correct acceleration on her particle. This local result of Alice's will generate a result identical to the "nonlocal", Bell-violating result.

2. The Locality of de Broglie-Bohm Trajectories for Massive Singlets in Aligned Fields

When Alice and Bob's detectors are both at the same angle, $\theta_A = \theta_B$, then $s = 0, c = 1$ and the velocity law, like Hooke's law, reduces to a function of the relative distance coordinate only:

$$\frac{d(z_L - z_R)}{dt} = \frac{k^2}{1+k^2t^2}t(z_L - z_R) + \alpha t \left[2 - \frac{k^2t^2}{1+k^2t^2} \right] \cdot \tanh \left(\frac{\beta t^2}{1+k^2t^2} \frac{(z_L - z_R)}{2} \right). \quad (8)$$

Locality	Normalization	Detector Efficiency	S_{Bell}	std. dev.
Loc	singles	Efficient	-1.31946	$\pm .03652$
Loc	coincidences	Inefficient	-2.76893	$\pm .07086$
NonL	singles	Efficient	-2.77554	$\pm .03652$
NonL	coincidences	Inefficient	-2.76893	$\pm .07086$

TABLE I: Calculationally measured Bell parameters for various “computational” configurations.

Since we have shown that the Hooke (or Coulomb) type force law is not *actually* nonlocal, being an idealization of a more fundamental local continuum model, we can conclude that the entangled particle velocity law – at least when encountering aligned magnets – is also generating *local* trajectories, as long as the system is isolated. This is important because a recent paper²⁴ purports to show that “Bell Locality” imposes certain constraints on *any* hidden variable theory, such that it must then obey Bell’s inequality. Since we have just shown that the de Broglie-Bohm interpretation becomes a local model (and satisfies the constraints Norsen proposed for the case of aligned magnet settings), we have an obvious contradiction since we know that our interpretation is local and the de Broglie-Bohm model will reproduce the quantum mechanical results. The contradiction can be resolved by realizing that Bell’s inequality implicitly requires hidden variable theories to violate Heisenberg’s uncertainty principle¹⁷ – whereas de Broglie-Bohm can predict counterfactual results (*i.e.*, $A(a, \lambda, t)$ and $A(a', \lambda, t')$), but it can’t predict them simultaneously, but only $A(\frac{a+a'}{2}, \lambda, t)$ in such a case.

The fact that Alice’s velocity depends on c and s (in particular, on θ_B), makes this de Broglie-Bohm trajectory violate one of Bell’s assumptions,³⁶ that the measurement result, A , be written as $A(a, \lambda)$, and not as $A(a, b, \lambda)$, (or $A(a(t), b(t - \tau), \lambda)$). In the case that a Dirac wave has time to propagate from Bob to Alice, then there is no actual nonlocality, just a nonlinear sensitivity to (remote) initial conditions that is only revealed by entangled wavefunctions – indeed all of the physical phenomena that tout nonlocality only require entanglement in order to be understood, and we might refer to this as “contextuality at a distance”.

3. Computer Experiments

We have programmed an EPR experiment, using the Durt and Pierseaux velocity prescription to determine the motion of neutral silver atoms in a 30 cm magnet with a field gradient of 10^4 G/cm, and initial spread in the Gaussian wavefunction of 10^{-3} cm, so that $\alpha = 2.58 \cdot 10^5$ cm s⁻², $\beta = 5.17 \cdot 10^{11}$ cm⁻¹s⁻², and $k = 2.94$. The values in table I resulted from the tracking of 4000 pairs of particles, where new magnet settings were randomly chosen as each pair was launched (this means that even though the particles took a millisecond to traverse the magnet, and presumably as long to get to the magnet, we pretend that the speed of light is even slower, so that Alice and Bob are spacelike separated).

Starting with the bottom two rows of the table, if Alice and Bob use non-local knowledge of the other’s instantaneous magnet setting, and if there are no physics-induced detector inefficiencies, the coincidence rate will be the same as the singles rate in each detector, so that normalizing by singles (or coincidences) will result in a small standard deviation (0.036, third row); if there are physics-induced inefficiencies (see section IV below), then the number of coincidences will be about 1/4 the number of single events, leading to a standard deviation twice as large (bottom row), in both cases violating Bell’s inequality.

If Alice and Bob only use local knowledge of the other’s setting (*i.e.*, retarded boundary conditions), and if there are no detector inefficiencies, then they will calculate the correct position of their partner’s particle only half the time (on average) and the Bell parameter they measure will be less than 2 (1.3, but with a small standard deviation, first row). If there are processes that throw particles out of the beamline before getting to a magnet that has just been changed, then Alice and Bob will find their coincidence rate to be half their singles rate, but by normalizing to the coincidences, they will calculate a Bell parameter of 2.8 with a larger standard deviation (0.07, second row).

If Alice and Bob change their magnets without affecting the oncoming particles, and if the particles move at subluminal speeds ($v/c \sim 10^{-6}$) so that the Dirac wave outruns the particle, then Alice and Bob will be using the “effectively instantaneous” values of each others magnet settings. In such a case, they will measure a causal (local) violation of Bell’s inequality with a small standard deviation (identical to results in the third row).

IV. PHYSICAL BUT UNFAIR SAMPLING EFFECTS

In order to experimentally establish nonlocality with massive particles, it is necessary to switch the magnet settings on Alice and Bob's detectors in a random manner, and at a rate greater than the source emission rate. Clearly if two or more singlets are emitted before either detector changes settings, there will be a much higher percentage of "locally" describable events than the potentially non-local, for which Alice and Bob, using the old (wrong) settings, will predict the wrong event behavior. On the other hand, the settings should not change so frequently as to change while the particle is inside the magnet, nor should they change and change back again before the particle gets to the detector.

Using the Durt and Piersaux expressions with the time-lagged values of the other's magnet setting, we have shown, *via* computer experiments, that the Bell parameter drops to a value below 2.0 when Alice and Bob randomly pick a new detector setting after each singlet is launched (compared to $2\sqrt{2}$ when using the instantaneous values of both). If such an experiment could actually be performed, we would expect to see experiments measure values significantly less than 2 when (if) the "locality loophole" is closed, but $2\sqrt{2}$ if it is not.

However, we remind the reader that in such an experiment²⁵, the magnetic field will be of order 1-10 T or $\sim 10^5$ G (although Durt and Piersaux never mention the field strength), with spatial gradients of the field of order 10^2 T/m (10^4 G/cm). The magnet can be switched either mechanically, or by the equivalent of energizing two (opposite) poles of a quadrupole while de-energizing the other two poles. This change of magnetic field will result in the emission of a real electromagnetic (Maxwell) wave travelling down the flight-path toward the source of the particles (as well as in all other directions), and the wave will have a magnetic field in the direction of $a - a'$ (or $b - b'$). In other words, if Alice's settings for her magnetic field (\hat{B}) are $a = 0$ and $a = \pi/2$, then her $\Delta\hat{B}$ will point at $-\pi/4$ or $+3\pi/4$.

For the sake of clarity, let us assume that on all relevant timescales for this problem, it takes no time to switch the magnet from one setting to another. Thus Bob's full ΔB leaves the front end of his magnet and travels toward Alice, and $30/c$ seconds later (assuming that the magnet is $L = 30$ cm long), ΔB drops back to zero as the signal from the back end of the magnet leaves the front on its way to Alice. Within this virtual magnet volume that is sweeping from Bob to Alice, the gradients of the magnetic field are the same size as in the magnet $-\frac{dB_z}{dz} = 10^4$ G/cm.³⁷ (One might argue that electromagnetic fields should fall off as r^{-2} , making the magnitude of the field too small to have any effect. This would be true in free space, but for a neutral massive particle (*e.g.*, Ag) to cover a distance of order meters from source to detector, it will need to travel in an evacuated beamline, and those are usually made of steel; to the extent that this will act as a wave guide, the ΔB field strength will not attenuate.)

Since the Durt and Piersaux velocity formulae only depend on this derivative of the magnetic field, we can treat the EM wave washing over the particle in flight as if it were in a shorter magnet that would take $L/c = 10^{-9}$ s to traverse, and use Equations (6) to calculate that the particle will experience a transverse velocity *before* it even gets to the magnet, proportional to that shorter time. The transverse velocity for our typical silver atom will be about a part per million of what it would have been after leaving the real magnet, since the time in the real magnet is $\mathcal{O}(10^{-3}$ s) compared to the nanosecond "in" the EM wave, resulting in an insignificant effect.

Now consider a faster particle, say one moving 10^3 times faster, so that it only spends $\mathcal{O}(10^{-6}$ s) in the magnet (assuming for the sake of argument that the very small deflections will still be resolvable). The 30 cm long ΔB wave will still pass over the particle in 1 ns, so the transverse velocity will now be a part per thousand of those making the resolvable deflections. This too will probably not result in any lost particles.

However, if we now go for the ultimate neutral fermion, and send entangled pairs of neutrino's at the Stern-Gerlach apparatus, the neutrino will spend only twice as much time in the magnet as under the influence of the ΔB wave. This means the transverse velocity experienced between the source and the detector will be half that it would experience in the detector, and the deflection a quarter. Thus, for light particles moving at near light-speeds, one can expect that the ΔB field will kick any particle in flight entirely out of the experiment, deflecting it so much that it can no longer fit through any collimators at the entrance to the magnet.³⁸ Unfortunately, neutrino's have their own detector efficiency problems.

The simplest experimental signal of such a phenomenon would be to make absolute count-rate measurements, so that if Alice and Bob make no changes to their magnets, one measures a quiescent singles count rate, Q_1 somewhat larger than the quiescent coincidence rate C_2 (the latter being dependent on the square of the efficiency of nearly perfect detectors). If Alice and Bob now randomly switch their magnets once on average during the source-detector flight time, in the absence of this detector inefficiency, their coincidence rate should remain C_2 ; but if this effect is present, then the coincidence rate will drop to $C'_2 \sim C_2/4$, the singles rate to $Q'_1 \sim Q_1/2$, and these relative drops should be the same regardless of the efficiency of the individual detectors.

V. PHOTON EPR EXPERIMENTS

Tittel *et al.*^{26,27} performed an EPR experiment in Geneva that attempted to close the locality loophole by arranging that the photon in one arm of the experiment went through a beam-splitter into one of two analyzers. As the authors point out, a hidden variable could determine into which one of the two it would go, preserving locality, since the setup was static. The authors dismiss this, arguing that a quantum-mechanical random number generator (the beam splitter) is the best that can be imagined. Given that we have argued that entanglement is the underlying causal explanation for the violations of Bell’s inequalities, and entanglement relies on the quantum (Dirac) fields that permeate space, it requires no additional stretch of the imagination to believe that the de Broglie-Bohm-ish behavior of the photons at the beam splitter is also determined by similar (Maxwell) fields that determine the Bell violations in the detectors themselves. We conclude that this experiment has a gaping locality loophole.

The Innsbruck experiment of Weihs *et al.*²⁸ was designed to close the locality loophole, and did so if the assumption of fair sampling is valid. In this experiment, electro-optical modulators were used to rotate the polarization of the photon before it entered the polarizing beam-splitter/detector. These modulators are in some sense the analog of the magnetic field in the Stern-Gerlach apparatus, although no deviation of photon trajectories occurs inside the devices. If a detector loophole were the only flaw in this experiment, then one would have to rely on some property of the modulators, that, for example when switched, induces a loss of total-internal-reflection in the fiber-optic cable, so that photons are lost from the fiber if the switch is made. Although it is highly unlikely that such an effect would have gone unnoticed, it would be interesting to know if the coincidence counting rates vary when the modulators are turned off during the course of a setting measurement (*e.g.*, $\langle AB' \rangle$), compared to when the modulators switch between each coincidence event.

Given the nature of our local de Broglie-Bohm trajectories for massive particles, there is no conceivable way such a model could account for the spacelike separated EPR results without a detector loophole. On the other hand, there is no de Broglie-Bohm prescription for photons¹⁶ (p. 541), and it may be that the Clifford-algebra model of Christian¹⁰ may be a more correct idealization in this context than our fermion model, in which case the Innsbruck data may also be explainable by a local model.

VI. CONCLUSION

A. The Meaning of Nonlocality

The controversy over the EPR paradox has depended on ambiguous concepts of locality and contextuality. EPR were wrong (and Bohr was right as our de Broglie-Bohm expressions show) in that Alice’s measurement result *can* depend on the measurement that Bob makes, if the two particles are entangled. EPR would have been right to argue that if there is such a distance effect, it must be time-retarded in order to be causal (we thus predict that perfect photon detectors will show obedience to Bell’s inequality in Innsbruck-type experiments): time-retarded “action at a distance” is *not* nonlocal, but it is contextual.³⁹

Because EPR rejected contextuality at a distance, and equated that rejection to locality, Bell and others’ derivations of inequalities, as part of mathematically translating “no context at a distance” into $A(a, \lambda)$ and $B(b, \lambda)$, have implicitly assumed that there can be no “local contextuality” either, so that even if $A(a, \lambda)$ is measured, $A(a', \lambda)$ is assumed to exist, and is used in formulae in a manner to suggest that it *could have been measured* at the same time and place, thereby limiting their inequality to hidden variable theories that violate Heisenberg’s Uncertainty principle. If hidden variable theories are restricted by assumption to modeling commuting operators, it is not surprising that they fail to model non-commuting ones; Christian’s Clifford algebra¹⁰ provides a counterexample of a local model with non-commuting properties.

B. Résumé

The “manifestly nonlocal” equations of motion, be they Hooke’s or de Broglie-Bohm’s, are valid only as long as the system they are intended to model is isolated, which means only as long as no one *tries* to signal by grabbing one of the no-longer-isolated parts. Any attempt to change the boundary conditions destroys the assumption of isolation, invalidates the equation(s), and moots the point of whether the equation can signal or not. We have also seen that two-body idealizations are approximations of an underlying local continuum model; Hooke has his Navier-Stokes, but there is still work to bring QED/QCD to an equally satisfactory footing to underlie the de Broglie-Bohm interpretation of Schrödinger/Dirac.

By analogy with Hooke's Law, we have developed a local interpretation of entangled trajectories given time independent wavefunctions. We have also shown that this interpretation, for time dependent wavefunctions, can be made truly nonlocal by using, unjustifiably, non-retarded values of the wavefunctions. This makes the locus of de Broglie-Bohm nonlocality reside in superluminal Dirac waves, which is patently absurd. On the other hand, we can calculate local, causal trajectories by using appropriately time-retarded wavefunction(s) in the "calculation" of the velocities for these particles, making this the first local model (to our knowledge) that is physics-based and not artificially contrived. Noting that the Dirac waves for massive particles are such that any changes in those waves (and their entanglement) can only move at the speed of light while the massive particles guided by them move at subluminal velocities, we have concluded that it will be very difficult to close the locality loophole for such particles²⁹, but that if it is closed, we predict they will satisfy Bell's inequality.

Finally, we have also shown that act of changing the magnetic fields generates an EM wave that can affect the oncoming particles and could, for relativistic particles, kick them out of the detector, thereby creating a detector loophole in exactly the conditions that would otherwise close the locality loophole.

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 - ³² We will confine our discussion to massive particles, for which there is a well-defined de Broglie-Bohm interpretation based on the first-quantized Schrödinger and Dirac wavefunctions.
 - ³³ Squires' work^{15,197} retarded positions in time *independent* wavefunctions.
 - ³⁴ This sneaks in when one tries to avoid "contextuality" without making a distinction between distant contexts (Bob's settings affecting Alice's results) and nearby ones (Alice not being able to make both settings simultaneously).
 - ³⁵ For example, even if a person could understand two questions that were asked of him simultaneously, there is no way that he could answer them simultaneously. Such psychological "measurements" (*i.e.*, "trick questions") can be incompatible, such that

different answers result depending on the order the questions are asked, and could require modeling with non-commutative models.

- ³⁶ Examining Wigner’s derivation of Bell’s inequality³⁰, it is clear that local time-delayed knowledge of one’s partner’s settings is explicitly excluded in his proof as well.
- ³⁷ Although Durt and Piersaux assume for simplicity of exposition that no other component of \vec{B} has any gradients, this is not physical – in order for $\nabla \cdot \vec{B} = 0$, it must be the case that at least $\frac{dB_x}{dx} = -\frac{dB_z}{dz}$ if there is no gradient in the direction of motion, but more generally, $\frac{dB_x}{dx} \approx \frac{dB_y}{dy} \approx -\frac{1}{2}\frac{dB_z}{dz}$. In this case, as long as *any* electromagnetic field is traveling, the magnitude of the spatial gradients in any direction will be comparable to any other direction.
- ³⁸ The fact that a neutrino’s mass is $\mathcal{O}(1)$ eV compared to $\mathcal{O}(10^{11})$ eV for Ag^{108} means that α in Equation (7a) will increase a lot more than the time in the magnet will decrease – $(\alpha\Delta t)_\nu \sim 10^5(\alpha\Delta t)_{\text{Ag}}$ – assuming the magnetic moments are comparable; alternatively, while a silver atom at $v/c = 0.99$ would require a magnet thousands of kilometers long to generate the same deflections if the mass in the expression for α (Equation (7a)) goes over to γm_0 (so that α drops by a factor $\mathcal{O}(10^{-2})$ and Δt drops by $\mathcal{O}(10^{-6})$), it would avoid being kicked out of the beamline before the detector by a changing field.
- ³⁹ EPR assumed that either QM was incomplete, or that the physical quantities corresponding to non-commuting operators weren’t simultaneously real; they did not distinguish³¹ (p. 780) between the measurement result’s reality – the positive or negative deflections in a magnet (“...we must consider the quantity P [*i.e.*, $\sigma_x = \pm 1$] as being an element of reality...” – and the “hidden” properties of a particle that caused such deflections, even though they referred earlier³¹ (p. 777) to “...an element of physical reality *corresponding* to this physical quantity” [my emphases]. A de Broglie-Bohm interpretation of Bohm’s spin example would distinguish between the left or right value of the hidden position coordinate, x , which results in $\sigma_x = \pm 1$, and the up or down value of the vertical position coordinate, y , that results in $\sigma_y = \pm 1$. A magnetic field cannot be devised that measures both deflections simultaneously, but both hidden positions exist (simultaneously) even if only one is relevant to a given measurement.